Project Report: Using Pair Correlation Function of a Bose-Einstein Condensate to Evaluate Its Temperature

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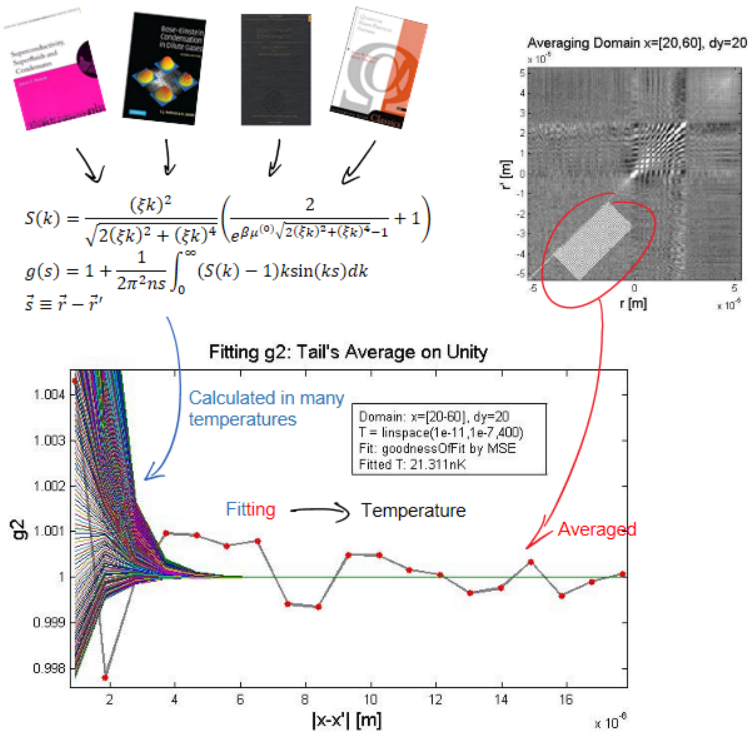
This document is intended for students.



Abstract

Prof. Steinhauer researches a Black Hole analogue he creates for phonons in Bose-Einstein Condensate (BEC). My goal in this project was to evaluate the temperature of the condensate outside the black-hole using the experimental data of pair correlation function $g(|\vec{r} - \vec{r}'|) \equiv g2$.

The project involved experimental work and much self-learning of theoretical background in quantum field theory and quantum many-particle systems, ideal BEC and Bogoliubov theory for weakly-interacting BEC. I consulted with Jeff on a weekly basis or whenever I needed help or direction, and I am most satisfied with the theoretical knowledge and the experience I gained with his guidance.

This project gave me an enlightening glimpse to the field of condensed matter. I would definitely recommend other students to do projects in fields which fascinate them, especially ones they don't know too well !

The Learning Process – Understanding What I Do

At first Jeff gave me experimental data of $g2$, and two relevant relations (written on the front page). I actually thought this was only the introduction of the project and it would take me two weeks or so to finish it and continue to the main work. In a week I indeed processed the experimental data. However, the next stage – calculating the theoretical $g2$ and fitting it with the experimental one, did not take me another week – but a whole semester of much effort, learning, interest and satisfaction.

I didn't want to calculate $g2$ by using relations I didn't understand how to derive. At first I thought a bit of reading will suffice. I found a great book with everything I needed. However, I couldn't understand it – it was too complicated for me at that time. I first read about an ideal BEC from an elementary book[1], then I proceeded to read a very detailed book[2] about BEC written in a higher level – and only then I returned to the first book[3]. Jeff gave me the key to understand it – I needed to learn second quantization[4] ! After learning what I needed I could crack it and advance, and finally I succeeded deriving the relations I needed by myself.

Theoretical Background

Bose-Einstein Condensation of ideal gas – with no interaction between the particles – is not very hard to solve. However, the problem of interacting particles is not trivial, and cannot be treated as a simple perturbation of the ideal problem. Bogoliubov theory (1947) is a set of assumptions and approximations which enable to solve the problem of BEC of weakly-interacting (or dilute) gas. The dispersion derived: $\varepsilon(p)=\sqrt{\frac{gn}{m}p^2+\left(\frac{p^2}{2m}\right)^2}$, when $n=\frac{N}{V}$ and g is the coupling constant of the interaction.

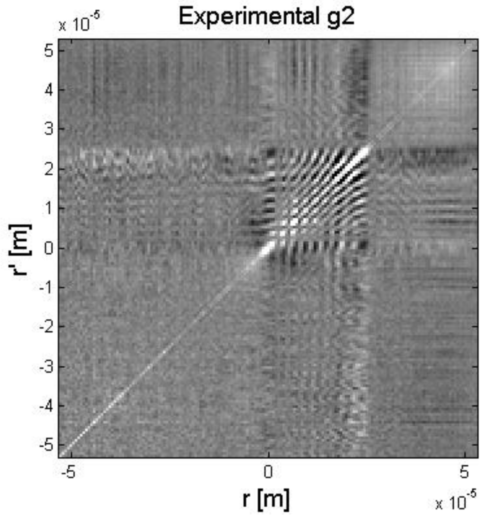
Linear response theory deals with the dynamic behavior of interacting many-body systems. The structure factor characterizes the scattering from the particles. Its zeroth moment – the <u>static</u> structure factor $S(\vec{k})$ – is related to the fluctuations of the density: $S(\vec{k})=\frac{1}{N}\left(\langle\rho_{\vec{k}}\rho_{-\vec{k}}\rangle-|\langle\rho_{\vec{k}}\rangle|^2\right)$, when $\rho_{\vec{k}}$ is the Fourier transform of the density operator $n(\vec{r})$. This is why it is also related to the pair correlation function $g(|\vec{r}-\vec{r}'|)$ by $g(s)=1+\frac{1}{n(2\pi)^3}\int\left(S(\vec{k})-1\right)e^{-i\vec{k}\cdot\vec{s}}d\vec{k}$, when $\vec{s}\equiv\vec{r}-\vec{r}'$.

After two months of learning I succeeded using Bogoliubov theory to derive the static structure factor of dilute BEC: $S(k)=\frac{(\xi k)^2}{\sqrt{2(\xi k)^2+(\xi k)^4}}\left(\frac{2}{e^{\beta\mu^{(0)}\sqrt{2(\xi k)^2+(\xi k)^4}}-1}+1\right)$, when ξ is the correlation (or healing) length. On length scales longer than ξ there is a correlation between the particles and they move collectively.

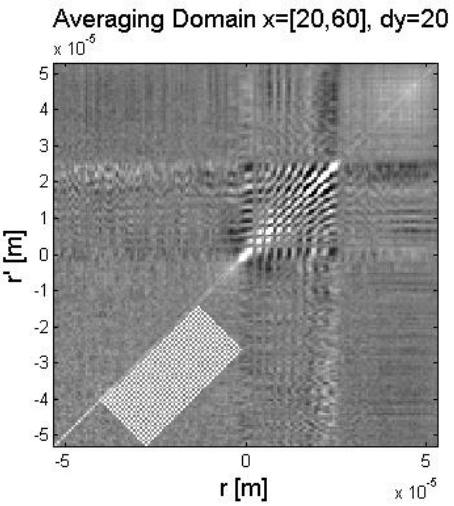
Finally, transforming to $g2$ was the easy part. Since the expressing I got for $S(k)$ is spherically symmetric in $\vec{k}$, integration yields $g(s)=1+\frac{1}{n(2\pi)^3}\int\left(S(\vec{k})-1\right)e^{-i\vec{k}\cdot\vec{s}}d\vec{k}=1+\frac{1}{2\pi^2 ns}\int_0^\infty(S(k)-1)k\sin(ks)dk$.

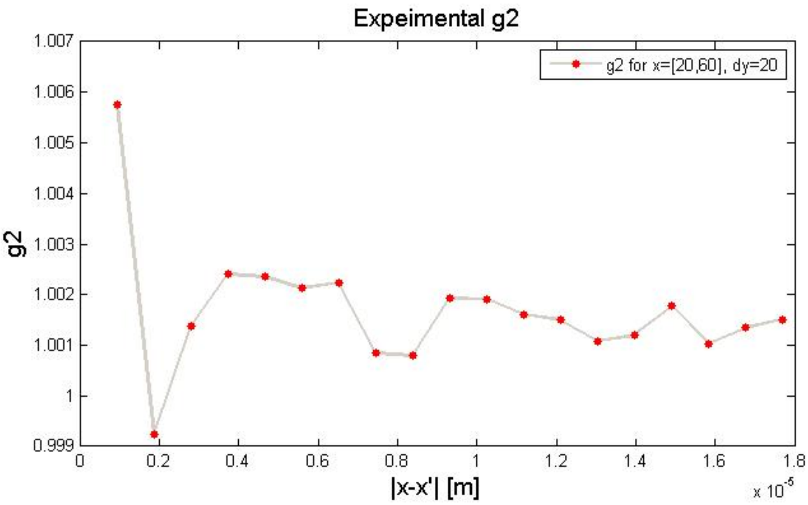
The Experimental Work

The pair correlation function $g(|\vec{r}-\vec{r}'|)$ measures correlation between two particles – by how much does the probability to find a particle at $\vec{r}'$ increase, given there is a particle at $\vec{r}$. This is the experimental data as an image – the brightness is the value of $(|\vec{r}-\vec{r}'|)$:



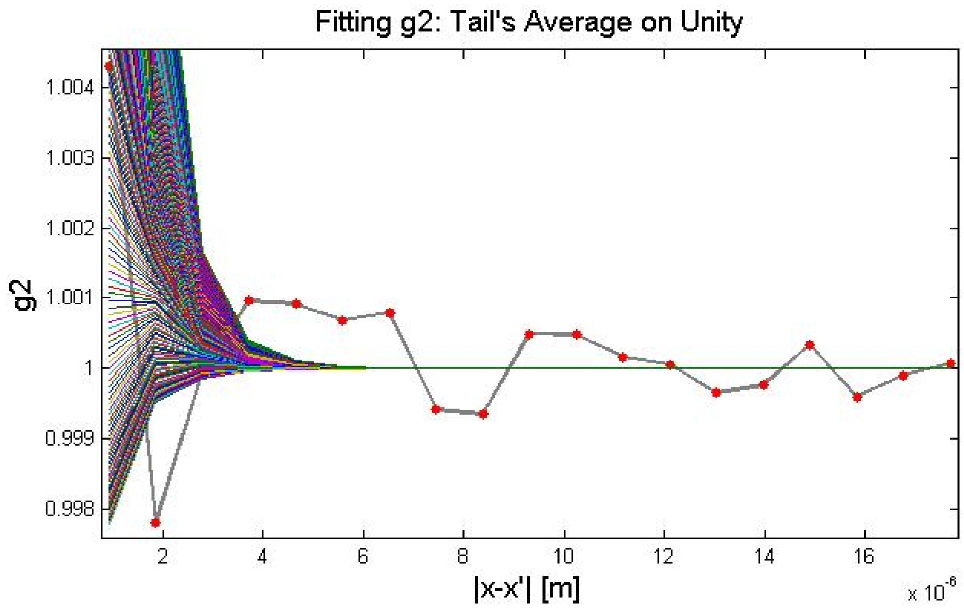
Of course $g(|\vec{r}-\vec{r}'|)$ is symmetric for $\vec{r} \leftrightarrow \vec{r}'$. I averaged the values of the function on a domain shown in white on the image below:



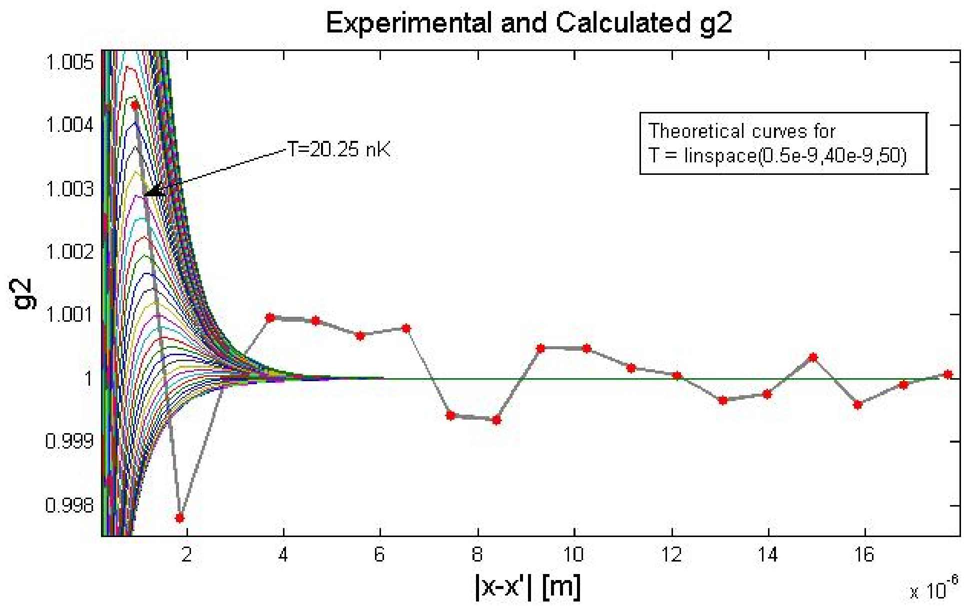


Then I calculated the theoretical $g2$ of different temperatures (the temperature is a parameter of $g2$). It was not so easy to me as $g2$ is not an analytic function but an integral transform of the static factor $S(k)$. The hardest theoretical challenged in the project was to succeed deriving the expression of $S(k)$ by using the Bogoliubov theory in second quantization.

Next I lowered $g2$ so its tail's average is on unity, because $g(|\vec{r}-\vec{r}'| \to \infty) \to 1$ so any deviation from unity is a systematic error. I calculated curves of g2 of different temperatures:



I measured the mean square error (MSE) of each $g2$ curve from the experimental $g2$. Jeff suggested to use the obvious peak the theoretical curves have in small distances to find the upper bound of 4 standard deviation units of the experimental $g2$ ("4-sigma criterion"):



This implies that $T \simeq 20\ \mathrm{nK}$ is the upper bound of the temperature of the BEC in this case. The lower bound cannot be determined in a simillar way because theoretical curves for every temperature cross the 4-sigmas below the mean, when $|\vec{r}-\vec{r}'| \to 0$.

In conclusion, we find that the temperature lies between 0 and 20 nK.

Dear reader, if you find the topic interesting consider checking http://physics.technion.ac.il/~atomlab/

References

1. Annett, J. F. (2004). *Superconductivity, Superfluids, and Condensates.*

2. C. J. Pethick, H. S. (2008). *Bose-Einstein Condensation in Dilute Gases.*

3. L. Pitaevskii, S. S. (2003). *Bose-Einstein Condensation.*

4. John W. Negele, H. O. (1998). *Quantum Many-particle Systems.*

Appendix: Main Code (Matlab)

```
clear;
clc;
load('analyze1 25022014B.mat');

%% Viewing Jeff's G2
figure;
%imagesc( xVect, xVect, n1DxiPrime * g2unfiltered(:,:,7), [ -scale1024
scale1024 ])
imagesc( xVect, xVect, 200 * g2unfiltered(:,:,7), [ -scale1024 scale1024 ])
    % The factor 200 is just to brighten the image (it was originally
    % n1DxiPrime)
colormap(gray);
set(gca,'YDir','normal');
axis image;

%% Averaging code
Domain=g2unfiltered(:,:,:);
x1=20; %lower limit
x2=60; %upper limit
dy=20; %WARNING: too big dy can exceed array bounds or enter black hole
domain

dr=dx*sqrt(2); % The averaging is done along the digonal of the pixels, not
their side - so its dx*sqrt(2)
r=1*dr:dr:(dy-1)*dr;

for y=1:dy-1
    temp=0;
    for x=x1:x2
        temp=temp+g2unfiltered(x,x+y,7);
        Domain(x-y,x+y,7)=0.1;
    end
    g2_exp(y)=temp/(x2-x1); %y starts from 0 and array index starts from 1
end

g2_exp = g2_exp+1; % because g2unfiltered = g2-1

%% Viewing averaging domain
figure;
%imagesc( xVect, xVect, n1DxiPrime * Domain(:,:,7), [ -scale1024 scale1024
])
imagesc( xVect, xVect, 200 * Domain(:,:,7), [ -scale1024 scale1024 ])
colormap(gray);
set(gca,'YDir','normal');
axis image;

%% Plotting g2_exp
%plot(r*dx,G2_Oz,'b');
createfigure2(r,g2_exp);

%% Calculating theoretical g2
xi = 2*sqrt(h_bar^2/(2*m*mu)); % correlation length. The factor 2 is because
of the trap potential
n = nPeak/2; % density. The peak is in the center of the trap so Jeff said I
can divide by 2

%T = [20e-9,20e-8,30e-10];
T = linspace(1e-11,1e-7,400);

s = 1*dr:dr:(dy-1)*dr;
%g2_calc = NaN(length(s),length(T));

for l = 1:length(T)
     for j = 1:length(s)
        Sk = @(k) ( ((xi*k).^2)./ sqrt(2*(xi*k).^2+(xi*k).^4) ).*(2./( exp(
mu*sqrt(2*(xi*k).^2+(xi*k).^4)./(kB*T(l)) ) -1) + 1);
```

```
        Integrand = @(k) (Sk(k) - 1).*k.*sin(s(j).*k);
        g2_calc(l,j) = 1 + (1/(2*pi^2*s(j)*n)) * integral(Integrand,0,1e8);
    end
end

%% Fitting experimental and calculated g2
g2_exp_lowered = g2_exp - 0.00144;

% figure
% plot(r, g2_exp_lowered, s, [g2_calc]);
createfigure_exp_and_calc(r, g2_exp_lowered, s, [g2_calc]) %check: hotter g2
is higher

for l = 1:length(T)
    fit(l) = goodnessOfFit(g2_exp_lowered,g2_calc(l,:),'MSE');
end

figure(3)
plot(T,fit,'.')
hold on
```