Sound propagation in a fluid in a cylindrical trap.

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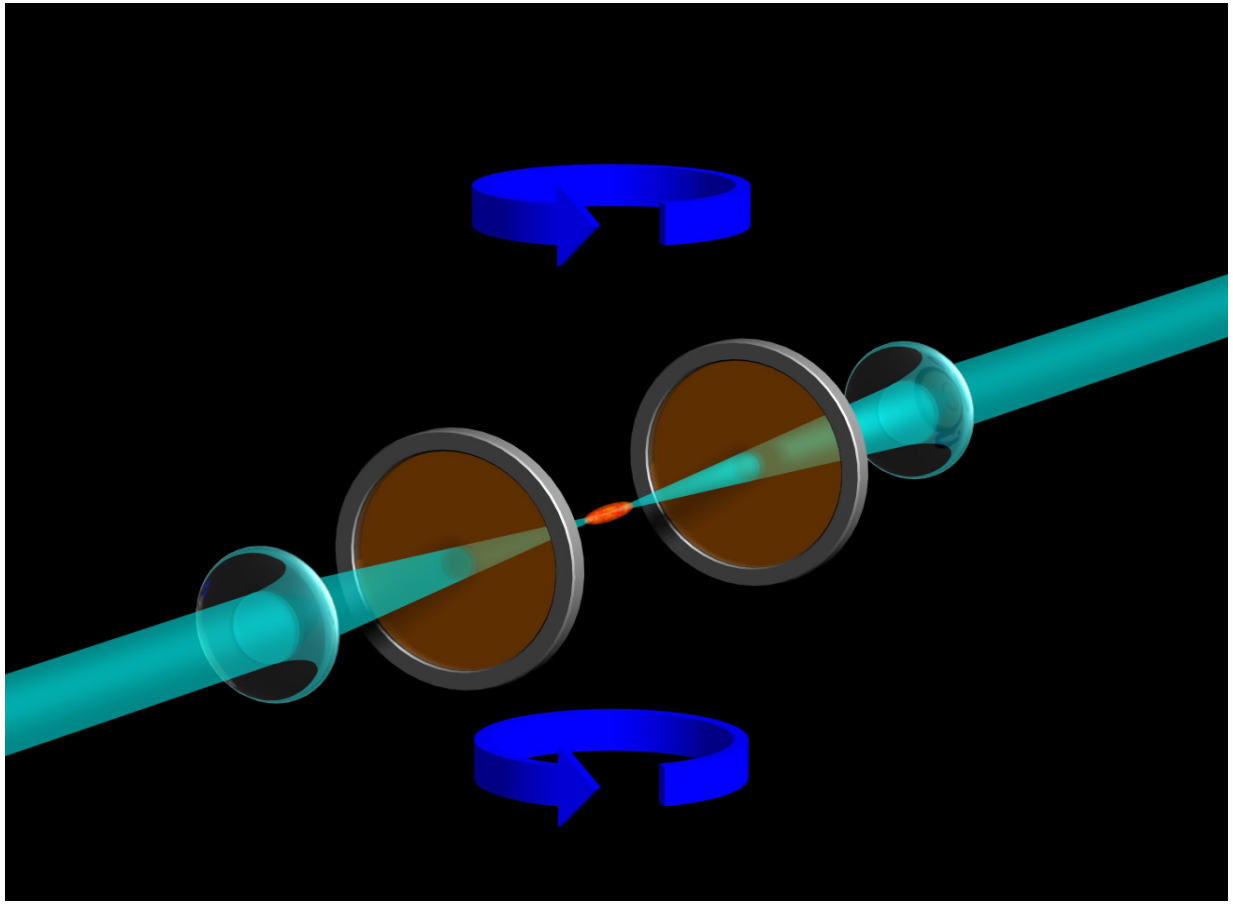
Introduction

Sound propagation in normal fluid and superfluid

Fermions in an optical trap (Duke University)



Sound waves in an uniform normal fluid

$$\text{Linearized equations of hydrodynamics:}$$

$$\rho \partial_{\mathrm{t}} \mathbf{v} = -\nabla \delta P$$

$$\partial_{\mathrm{t}} \delta\rho = -\rho \nabla \cdot \mathbf{v}, \quad \partial_{\mathrm{t}} \delta s = 0$$

$$\partial_t^2 \delta\rho = \left(\partial P / \partial \rho\right)_s \Delta \delta\rho$$

$$\delta\rho \propto e^{i(qz - \omega t)}$$

$$\omega^2 = q^2 c_a^2, \quad c_a^2 = \left(\partial P / \partial \rho\right)_s \text{ - adiabatic sound.}$$

$$\omega \delta s = 0$$

Density response function

$$\rho\partial_t v = -\nabla\delta P - \rho\nabla\delta V / m,\ \delta V \propto e^{i(qz-\omega t)}$$

$$\delta\rho = \chi(q,\omega)\delta V.$$

Im χ can be measured in Bragg scattering experiments.

Exact relations:

$$f\text{-sum rule}: \chi(\omega\to\infty)\to -q^2\rho/m\omega^2.$$

Compressibility sum rule:

$$\chi \to \frac{\rho}{m}\left(\frac{\partial\rho}{\partial P}\right)_T \equiv \frac{\rho}{mc_T^2},\ q,\omega\to 0, \omega/q\to 0.$$

Density response function – a problem

$$\chi(q,\omega) = \frac{\rho}{m}\frac{q^2}{c_a^2 q^2 - \omega^2}$$

$$\chi(\omega \to \infty) \to -\frac{\rho}{m}\frac{q^2}{\omega^2} \textbf{ OK!}$$

$$\chi(\omega \to 0) \to \frac{\rho}{mc_a^2} \neq \frac{\rho}{mc_T^2}$$

Sum rule is violated!

A solution - zero frequency mode

Take into account thermal conduction :

$$\rho \partial_t v = -\nabla \delta P - \rho \nabla \delta V / m$$

$$\partial_t \delta\rho = -\rho \nabla \cdot v, \quad \partial_t \delta s = \left(\kappa / \rho T\right) \Delta \delta T$$

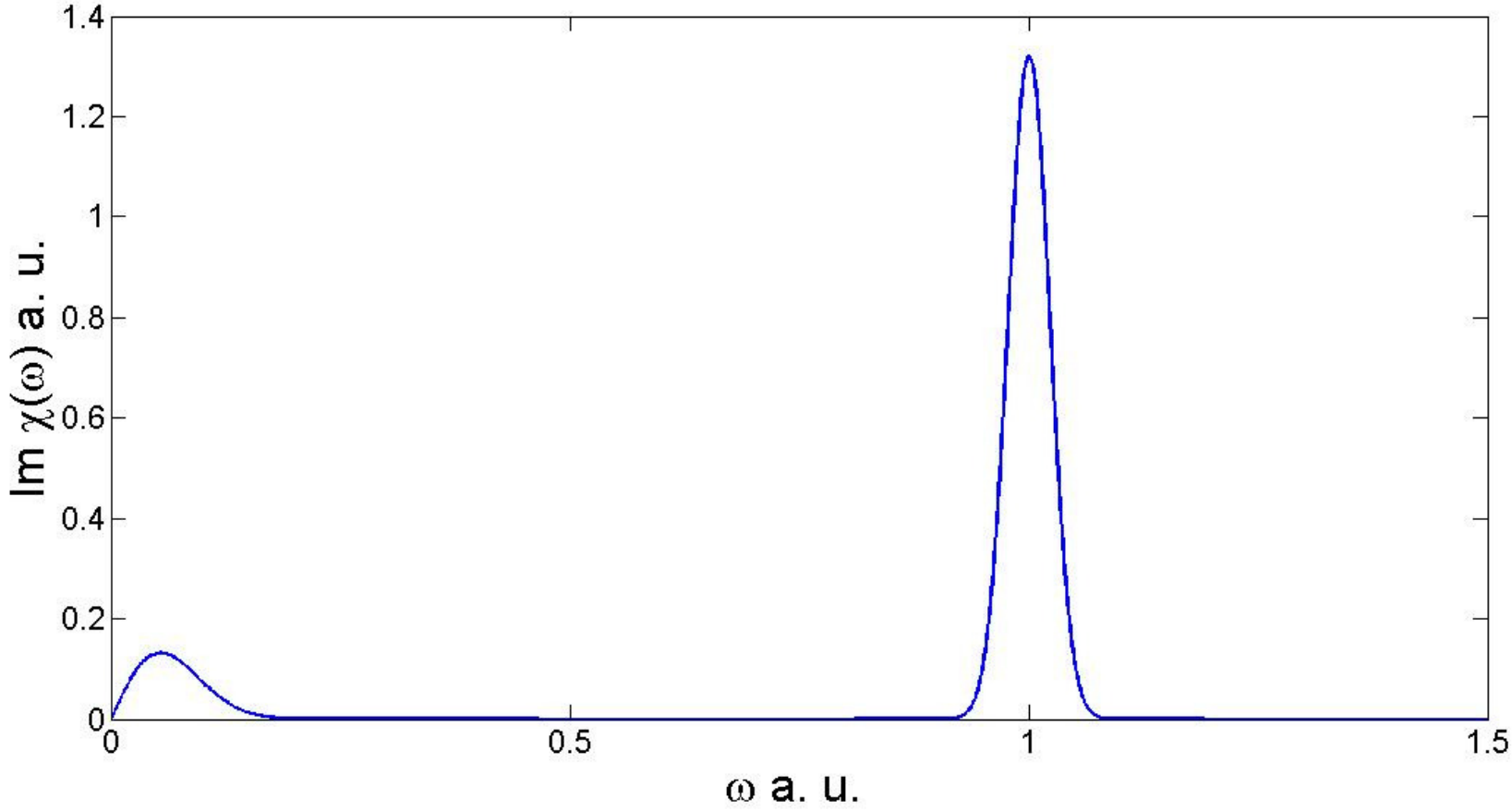
$$\text{I. } \left(\kappa / \rho T\right) q^2 << \omega << q c_a : \delta s \approx 0$$

$$\chi\left(\omega \to 0\right) \to \rho / m c_a^2.$$

$$\text{II. } \omega << \left(\kappa / \rho T\right) q^2 << q c_a : \delta T \approx 0$$

$$\chi\left(\omega \to 0\right) \to \rho / m c_T^2. \textbf{ OK!}$$

Schematic representation of Im χ for an uniform normal fluid.



Density response function and light scattering

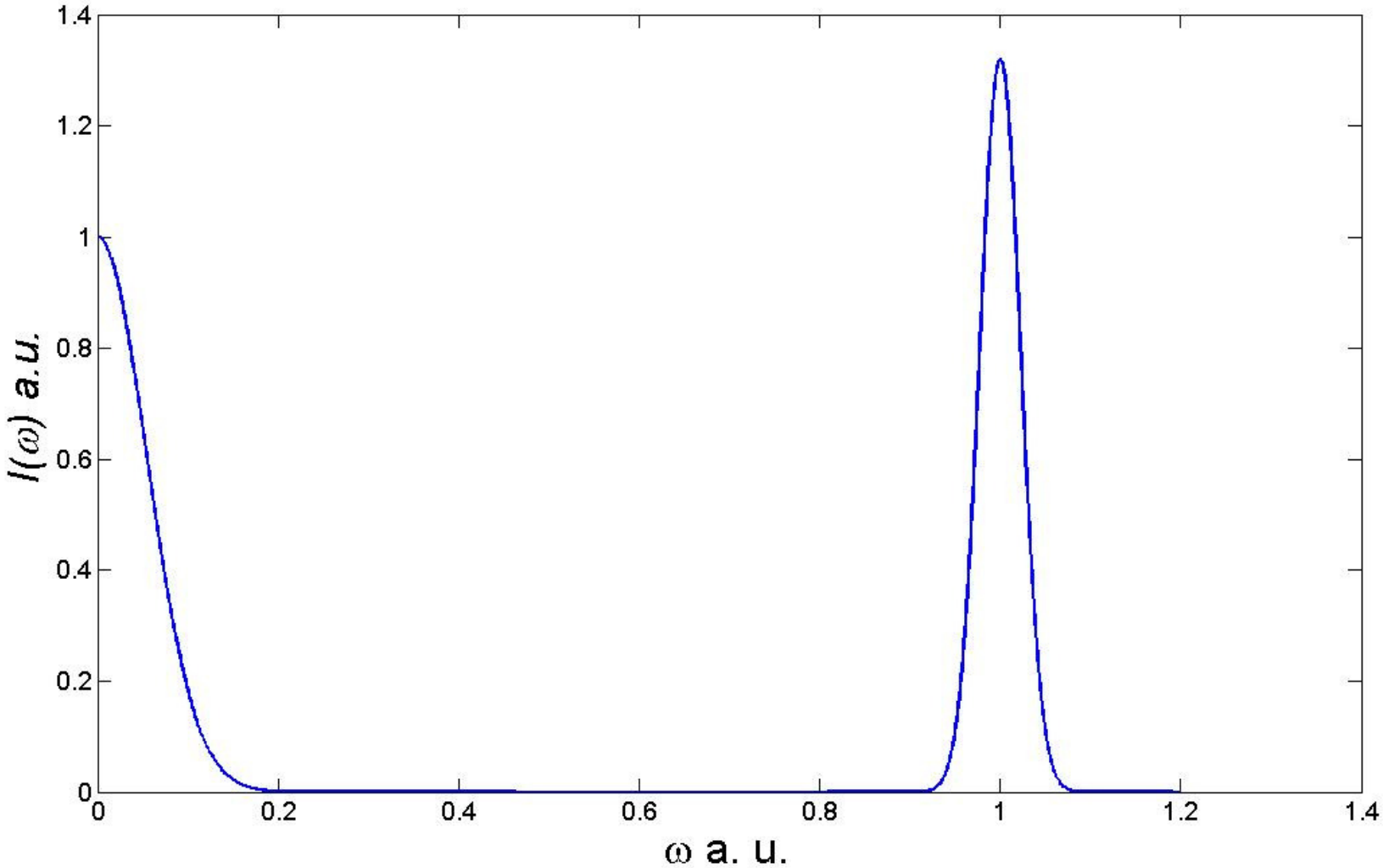
$\mathrm{Im}\ \chi(\omega, \mathrm{q})$ can be measured in the Bragg scattering experiments.

It also defines intensity I of light scattering in the fluid :

$$I \propto \frac{T}{\omega} \mathrm{Im}(\omega, q) \quad ,$$

ω and q are changes of frequency and wave vecor at scattering.

Schematic representation of intensity *I* of light scattering in an uniform normal fluid.



Superfuidity

Equations of the Landau hydrodynamics :

$$\rho = \rho_s + \rho_s, \mathbf{j} = \rho_s \mathbf{v}_s + \rho_n \mathbf{v}_n,$$

$$\partial_t \delta\rho + \nabla \cdot \mathbf{j} = 0, m\partial_t \mathbf{v}_s + \nabla \delta\mu = 0,$$

$$S = \rho s, \partial_t \delta S + S\nabla \cdot \mathbf{v}_n = 0,$$

$$\partial_t \mathbf{j} + \nabla \delta P = 0.$$

First and second sounds

Two sound modes -

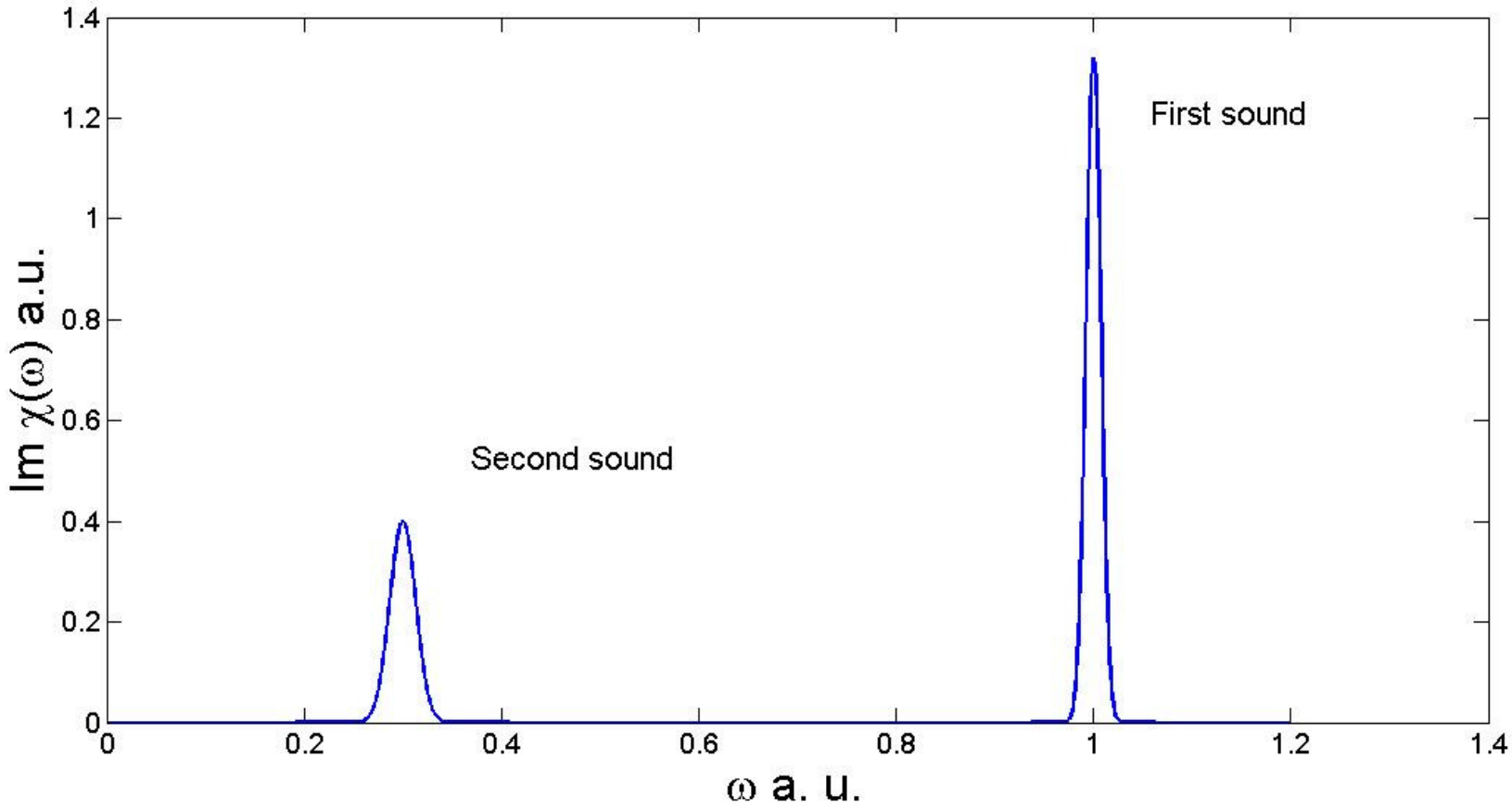
first sound and second sound.

If $\left(\partial\rho/\partial T\right)_P$ is small:

$$c_1^2 = \left(\partial P / \partial \rho\right), c_2^2 = \left(Ts^2 \rho_s / c\rho_n\right).$$

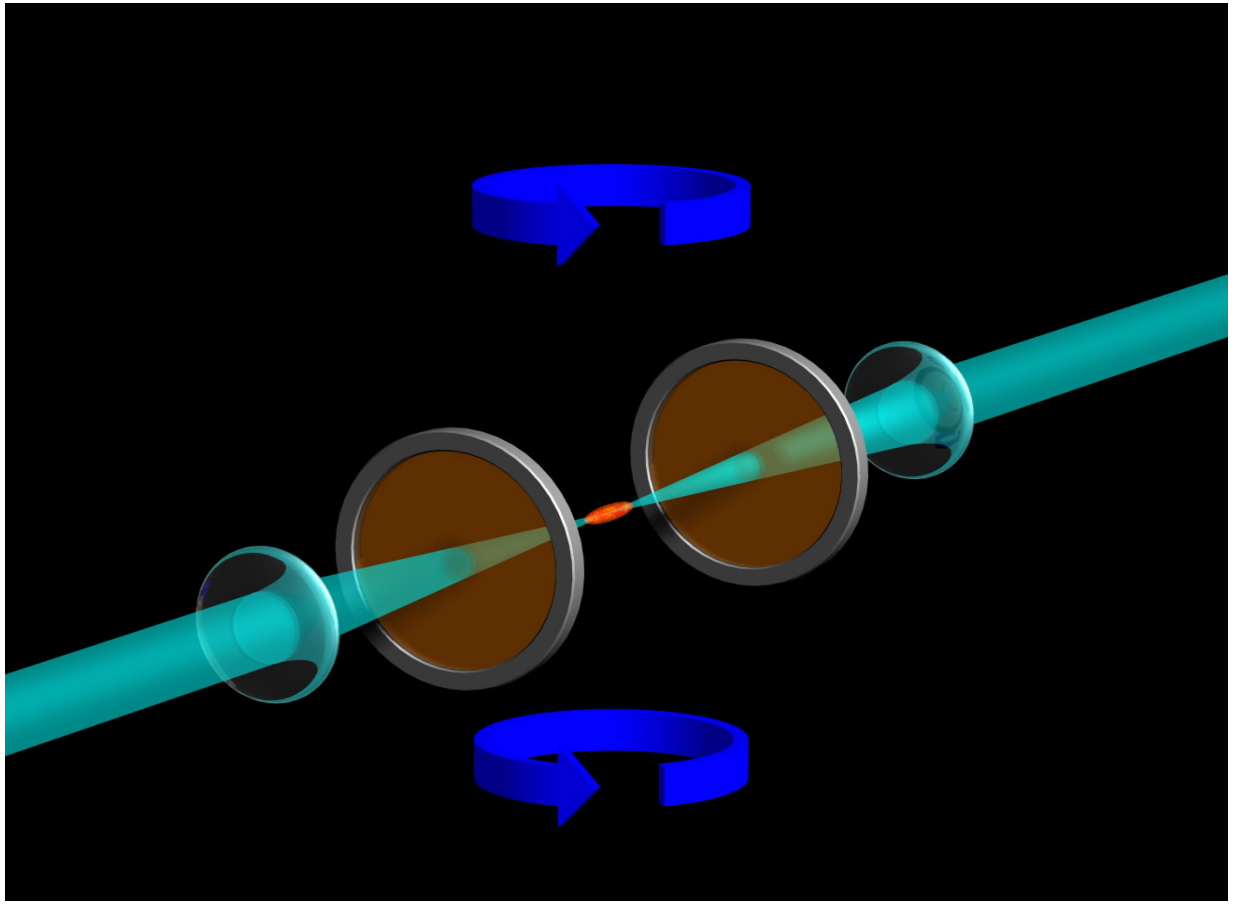
No problem with sum rules.

Schematic representation of Im χ for an uniform superfluid.



Sound propagation in a cylindrically trapped gas

Fermions in an optical trap (Duke University)



Gas in a tube

Propagation of sound in a tube

Hydrodynamics. Free path l is small: $l << \lambda$.

Viscous depth of penetration

$$\delta = \sqrt{\frac{\eta}{\rho_n \omega}}, \delta_T \sim \delta.$$

Two regimes :

I. Low frequencies : $\delta >> R$

II. High frequencies : $\delta << R$

Regime I, $\omega << \eta/(\rho_n R^2)$:

$$\nabla_\perp v \approx 0, \nabla_\perp \delta T \approx 0$$

A. Normal fluid :

$$v = 0 \text{ at } r_\perp = 0, v \equiv 0.$$

No propagating modes.

B. Superfluid :

$$v_n = 0, v_s \neq 0.$$

One propagating mode :

"4 - th sound", K. Atkins (1958).

Regime II, $\omega >> \eta/(\rho_n R^2)$:

One can neglect viscosity and thermal conduction.

Presense of the wall is not important.

A. Normal fluid :

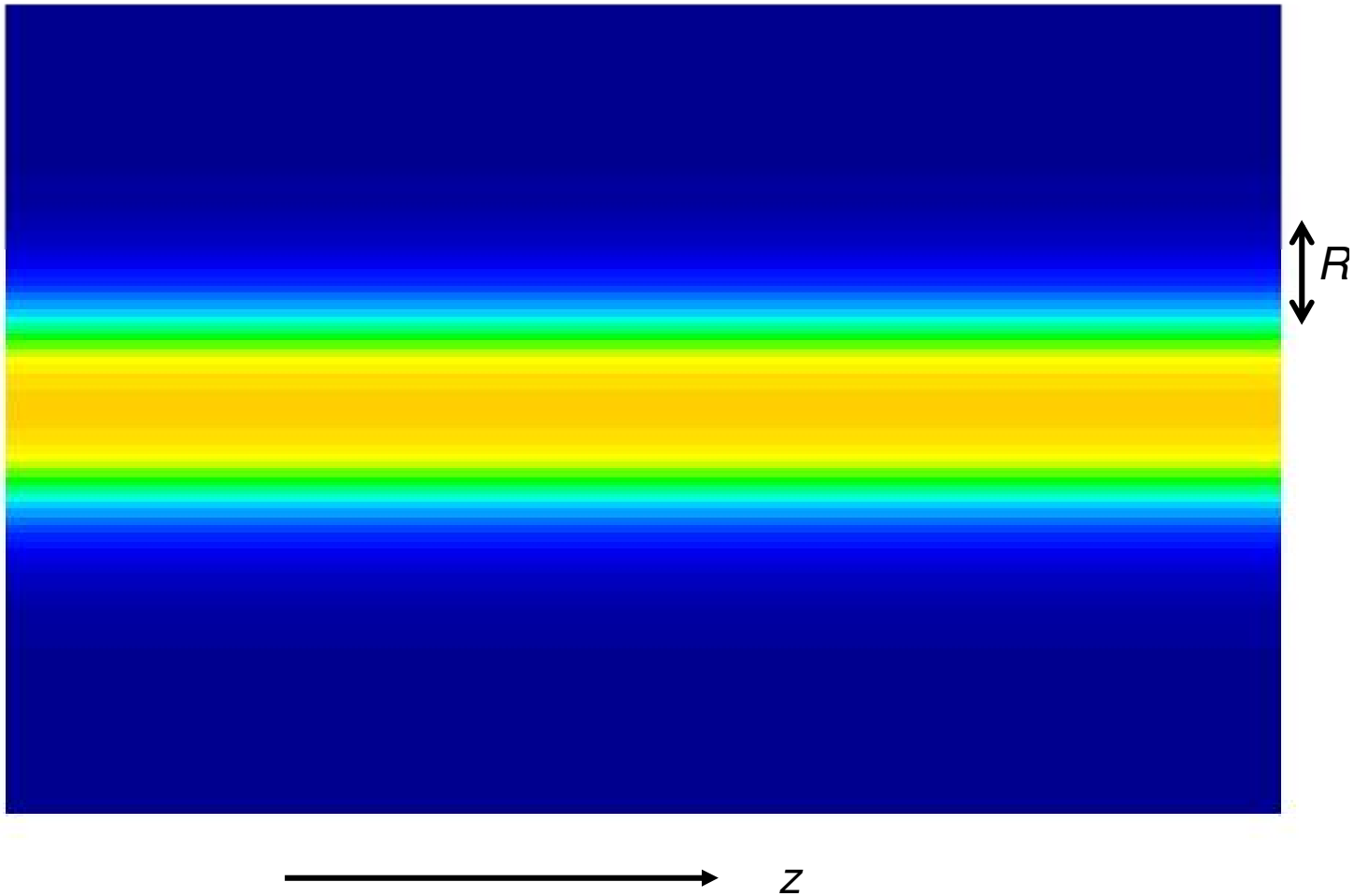
Usual adiabatic sound.

B. Superfluid :

First and second sounds.

Gas in a cylindrical trap

$$\text{Trapping potential}: U(r_\perp) = m\omega_\perp^2 r_\perp^2 / 2$$



Regime I, $\omega << \eta/(\rho_n R^2)$:

Like in a tube $\nabla_\perp v \approx 0, \nabla_\perp \delta T \approx 0.$

However trap is smooth.

No boundary conditions for v_n,

$v_n, \delta T \neq 0$ **but do not depend** on $r_\perp$.

A. Normal fluid :

One propagating mode.

B. Superfluid :

First and second sounds.

Regime II, $\omega >> \eta/(\rho_n R^2)$:

Like in a tube , one can neglect viscosity and thermal conduction.

A. Normal fluid :

Adiabatic sound.

B. Superfluid :

First and second sounds.

A classical ideal gas in a cylindrical harmonic trap.

$$\text{Regime II}, \omega >> \eta/(\rho R^2):$$

$$-m\omega^2 \mathbf{v} = \frac{5}{3} T \nabla[\nabla \cdot \mathbf{v}] - \nabla[\mathbf{v} \cdot \nabla U] - \frac{2}{3}[\nabla \cdot \mathbf{v}]\nabla U$$

$$U(r_\perp) = m\omega_\perp^2 r_\perp^2 / 2$$

A. Griffin, Wen - Chen Wu, S. Stringari (1997).

Boundray conditions:

$$\int_0^\infty \rho v^2 r_\perp dr_\perp < \infty, (v_\perp r_\perp)_{r_\perp \to 0} \to 0.$$

Exact solution - adiabatic sound

$$\mathbf{v} \propto \exp\left[i\left(qz - \omega t\right)\right]$$

$$\omega = qc_a,\ c = \sqrt{\frac{5T}{3m}}$$

$$v_z = Ce^{\xi^2/5},\ \xi^2 = m\omega_\perp^2 r_\perp^2 / \left(2T\right)$$

$$v_\perp = 0$$

T. Nikuni, A. Griffin (1998)

Density response function

$$U \to U + \delta V, \delta V \propto \exp\left[i\left(qz - \omega t\right)\right]$$

$$\delta\langle\rho\rangle = \chi(q,\omega)\delta V, \langle\rho\rangle = \int \rho dxdy$$

Sum rules for a trapped gas:

f - sum rule:

$$\chi(\omega \to \infty) \to -q^2\langle\rho\rangle / m\omega^2.$$

Compressibility sum rule:

$$\chi \to \left(\frac{\langle\rho\rangle\partial\langle\rho\rangle}{m\partial P}\right)_T = \frac{\langle\rho\rangle}{T}, \; q,\omega \to 0, \omega / q \to 0.$$

Density response function of trapped gas

$$\chi_a(q,\omega) = \frac{5}{9}\frac{\langle\rho\rangle}{m}\frac{q^2}{c_a^2 q^2 - \omega^2}$$

$$\chi_a(\omega \to \infty) \to -\frac{5}{9}\frac{\langle\rho\rangle}{m}\frac{q^2}{\omega^2}$$

$$\chi_a(\omega \to 0) \to \frac{1}{3}\frac{\langle\rho\rangle}{T} \neq \frac{\langle\rho\rangle}{T}$$

Sum rules are violated!

“Repair” of the “defect”

There are no zero - frequency mode or the second sound mode in a normal gas in a cylindrical trap .

Instead in a such gas a new kind of excitations exists.

At given q frequency ω runs continuous interval of values.

G. Bertaina, L. Pitaevskii, S. Stringari (2010).

Continuum spectrum mode

$$v_z = Ae^{\xi^2/4}[\sigma\cos\left(\sigma\xi^2\right) - \sin\left(\sigma\xi^2\right)/20]$$

$$v_\perp = -i\sqrt{\frac{3}{20}}\frac{q^2c_a^2 - \omega^2}{qc_a\omega_\perp}Ae^{\xi^2/4}\frac{\sin\left(\sigma\xi^2\right)}{\xi}$$

$$\sigma = \sqrt{q^2c_0^2/\omega^2 - 1}/4,\ c_0 = \sqrt{24/25}c_a$$

$$\omega/q < c_0$$

Contribution of the continuum spectrum mode into χ

$$\chi_c(q,\omega) = -\frac{\langle\rho\rangle q^2}{\omega^2}\frac{128}{\pi}\int_0^\infty \frac{x^2 dx}{\left[x^2+1-c_0^2 q^2/\omega^2\right]\left(x^2+1\right)\left(25x^2+1\right)}$$

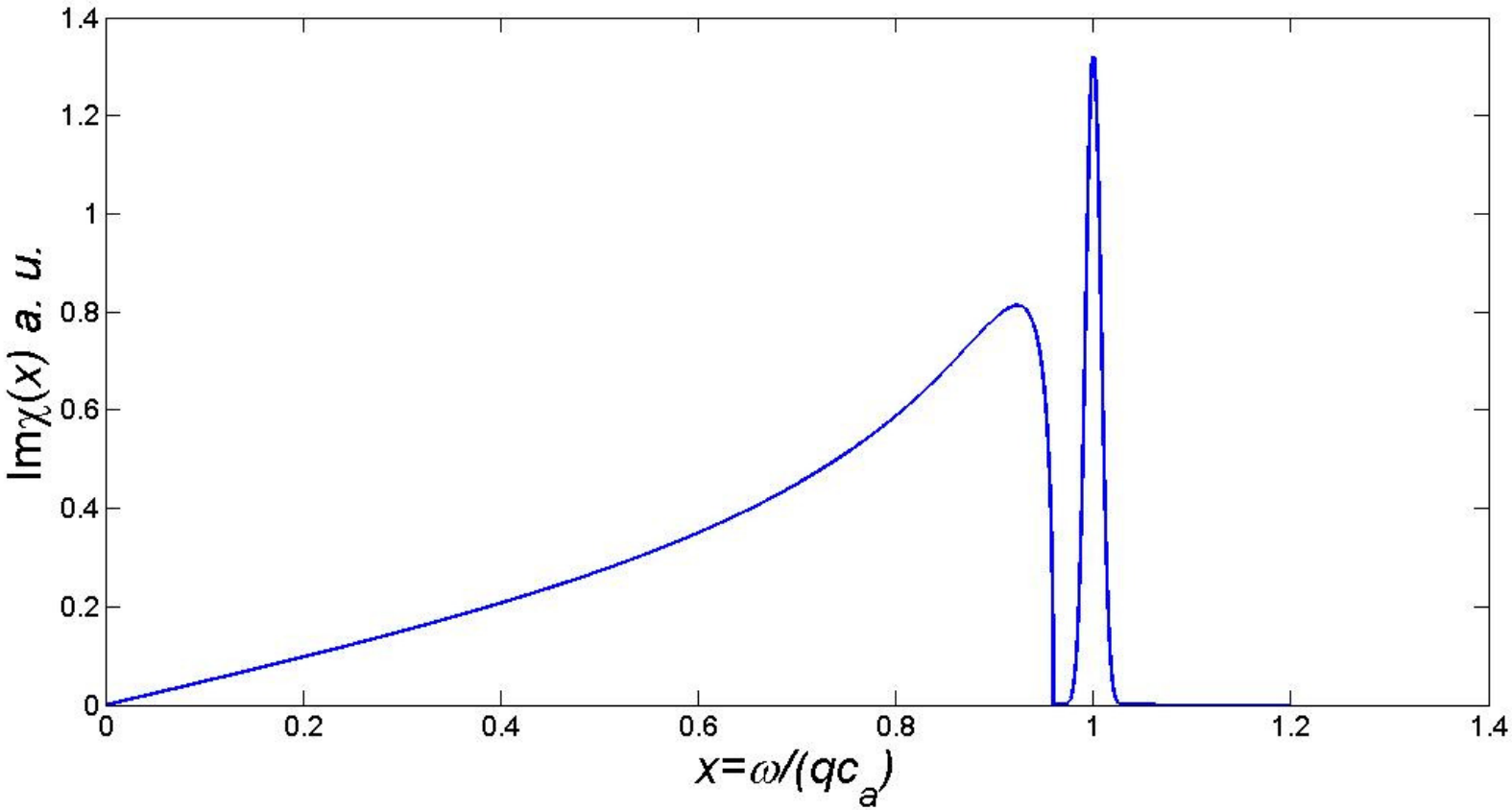
Function $\chi(q,\omega) = \chi_a(q,\omega) + \chi_c(q,\omega)$

satisfies sum rules relations.

$$\operatorname{Im}\chi_c(q,\omega) = \frac{4}{3}\frac{\langle\rho\rangle q^2}{mc_0^2}\frac{\omega\sqrt{c_0^2 q^2-\omega^2}}{c_a^2 q^2-\omega^2}$$

Can be measured at Bragg scattering experiment.

Schematic representation of Im χ for a cylindrically trapped classical gas



Linearized equations of Landau hydrodynamics in a cylindrical trap

$$\rho = \rho_s + \rho_n, \mathbf{j} = \rho_s \mathbf{v}_s + \rho_n \mathbf{v}_n,$$

$$\partial_t \delta\rho + \nabla \cdot \mathbf{j} = 0,$$

$$S = \rho s, \partial_t \delta S + \nabla \cdot \left(S \mathbf{v}_n \right) = 0,$$

$$\partial_t \mathbf{j} + \nabla \delta P + \rho \nabla U / m = 0,$$

$$m \partial_t \mathbf{v}_s + \nabla \delta\mu + \nabla U = 0.$$

Regime I, $\omega << \eta/(\rho_n R^2)$:

$$\nabla_\perp v_n \approx 0, \nabla_\perp v_s \approx 0, \nabla_\perp \delta T \approx 0.$$

$$qR << 1:$$

$$\nabla_\perp P + \rho \nabla_\perp U = 0$$

$$\nabla_\perp \delta P + \delta\rho \nabla_\perp U = 0 \rightarrow \nabla_\perp \delta\mu \approx 0$$

$$v_n, v_s, \delta T, \delta\mu \propto e^{i(qz-\omega t)}.$$

One must integrate equations with respect to dxdy.

Dispersion equation for sound

$$\text{Superfluid:}$$

$$c^4[m\langle\rho\rangle_\mu\langle S\rangle_T - \langle\rho\rangle_T^2] + c^2[2\langle\rho\rangle_T\langle S\rangle -$$

$$\langle\rho\rangle\langle S\rangle_T - \langle\rho\rangle_\mu m\langle S\rangle^2 / \langle\rho_n\rangle] + \langle\rho_s\rangle\langle\rho\rangle^2 / \langle\rho_n\rangle = 0.$$

$$\text{Normal fluid:}$$

$$c^2[m\langle\rho\rangle_\mu\langle S\rangle_T - \langle\rho\rangle_T^2] + [2\langle\rho\rangle_T\langle S\rangle -$$

$$\langle\rho\rangle\langle S\rangle_T - \langle\rho\rangle_\mu m\langle S\rangle^2 / \langle\rho\rangle] = 0.$$

$$\langle\ldots\rangle = \int\langle\ldots\rangle dxdy, \langle\ldots\rangle_T = \left(\partial\langle\ldots\rangle / \partial T\right)_\mu \text{ etc..}$$

An example: an ideal classical gas

$$c^2 = \frac{7}{5}\frac{T}{m}$$

This value is different from

one for a uniform gas :

$$c_a^2 = \frac{5}{3}\frac{T}{m}.$$

Fermi gas at unitarity

$$a \to \infty$$

$$P(\mu, T) = T^{5/2} H\left(\frac{\mu}{T}\right)$$

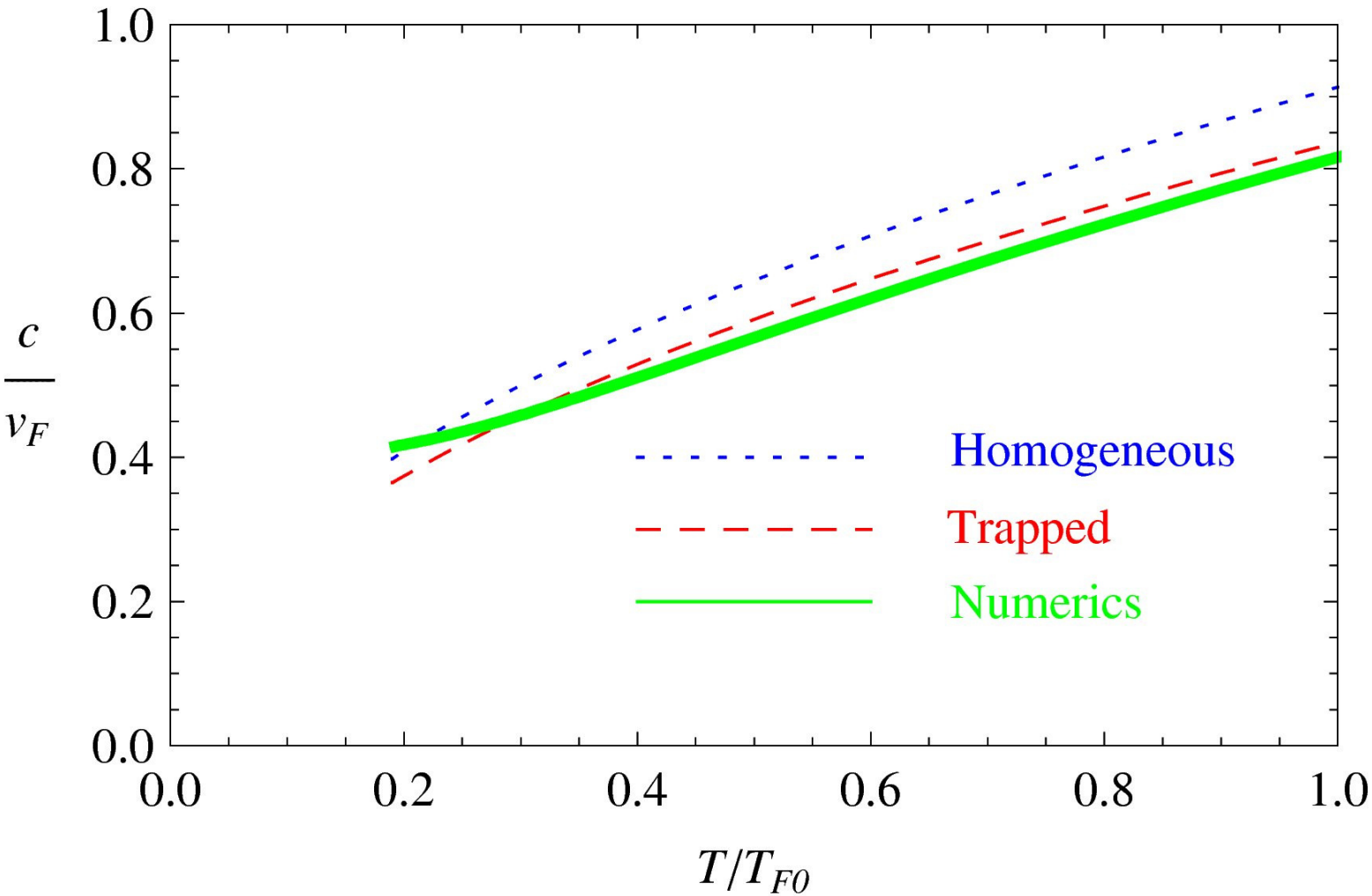
Was measured : Nascimbene et al., 2010.

$$\rho(\mu, T) = mT^{3/2} H'\left(\frac{\mu}{T}\right)$$

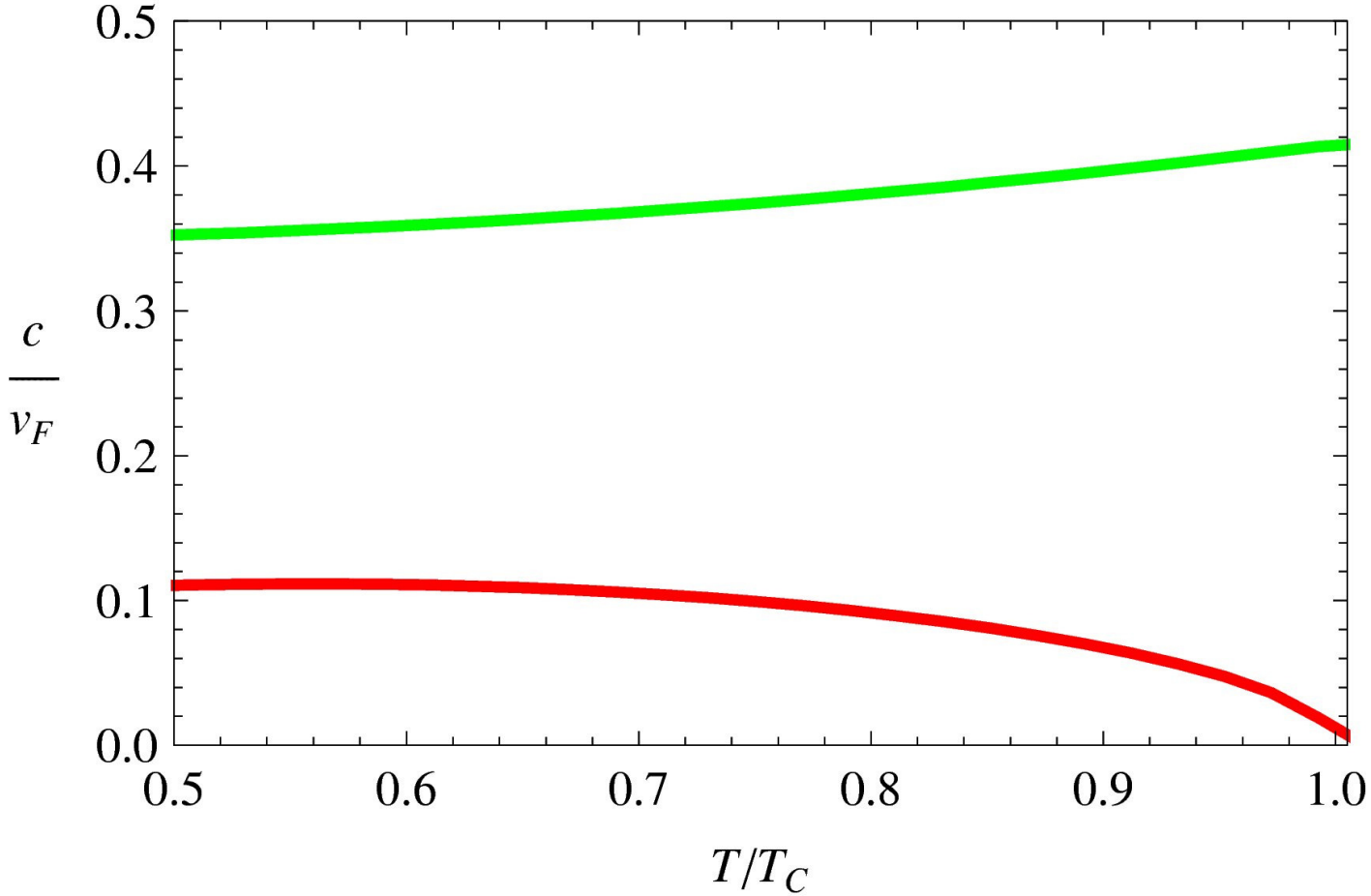
$$\rho_s(\mu, T) = mT^{3/2} \nu_s\left(\frac{\mu}{T}\right)$$

Was calculated : Fukushima et al., 2007.

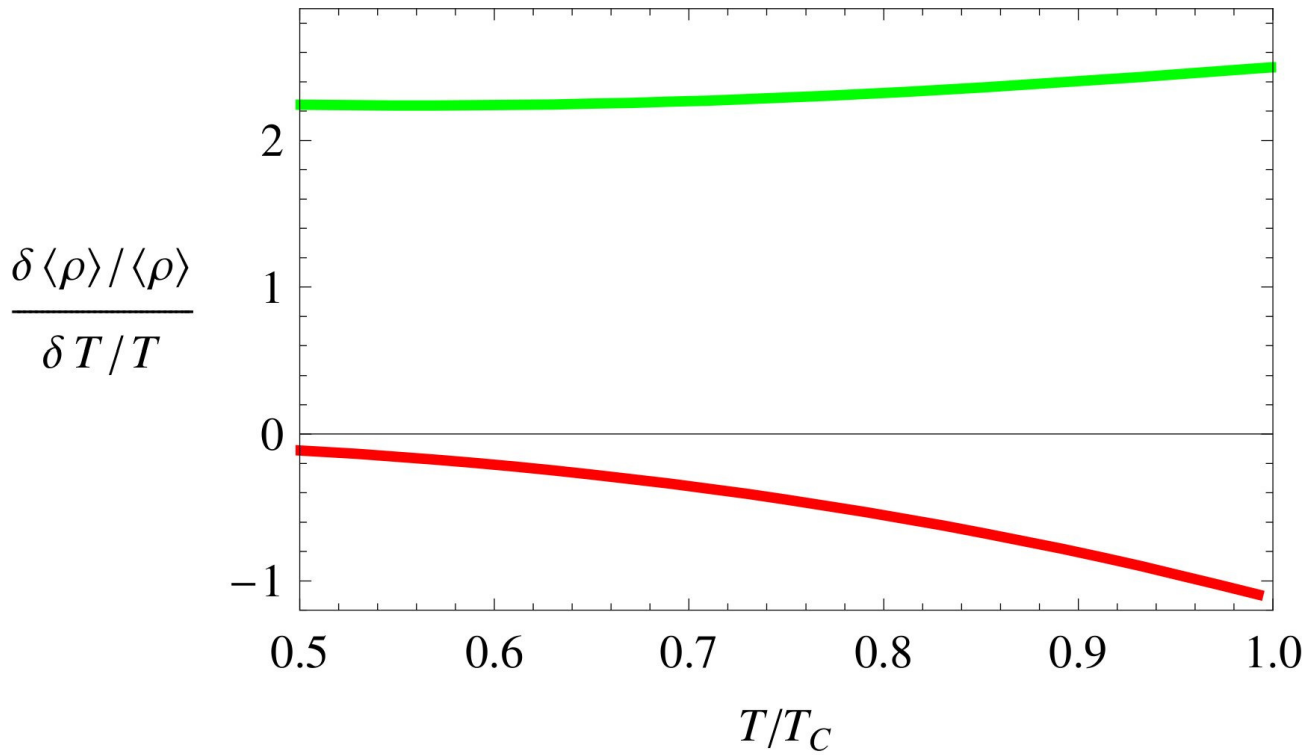
Velocity of sound at unitary Fermi gas above T_C in comparison with an ideal gas



Velocities of two sound modes at unitary Fermi gas below T_C



Density and temperature fluctuations



Ratio of contributions of two sound modes in Imχ